Master project:
Data Assimilation in Fusion Devices

INSTITUT de MATHÉMATIQUES de MARSEILLE

This internship offers an exciting opportunity to connect computational plasma physics with real experimental data. Your goal will be to make our plasma simulations "match" real measurements $f^{\exp}$ by directly integrating them into the numerical model using an adjoint-based method.

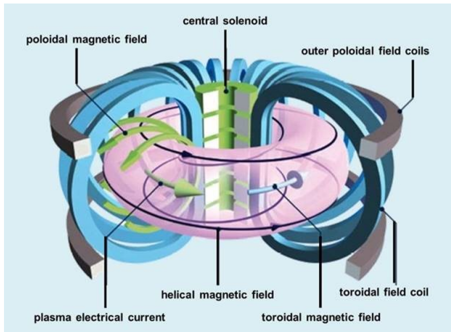
As a first step, you will work on data assimilation for the Vlasov–Poisson system:

$$\partial_t f + v\,\partial_x f + \frac{q}{m} E\,\partial_v f = 0, \qquad \partial_x E = 1 - \int f\,\mathrm{d}v. \tag{1}$$

The aim is to find the best possible initial state $f_0(x,v)$ so that the solution of eq. (1) reproduces the experimental observations. This leads to the following optimization problem with the cost functional

$$J = \frac{1}{2}\iiint_{t_0}^{T} \left(f(x,v,t) - f^{\exp}(x,v)\right)^2 \sigma_1(x,v)\,\sigma_2(t)\,\mathrm{d}t\,\mathrm{d}x\,\mathrm{d}v, \tag{2}$$

where $\sigma_1(x,v)$ and $\sigma_2(t)$ are weighting functions selecting the space–time region where measurements are available.



[Image courtesy of EUROfusion]

To solve the constrained optimization problem eqs. (1) and (2), you will use an adjoint optimization approach (see [1] for an introduction). The basic idea is:

- run the Vlasov–Poisson simulation forward in time to evaluate the cost functional,
- run an adjoint equation backward in time to compute how the initial condition should be improved.

This strategy has already been used, for example, to optimize the electric field E in [2, 3]. The exciting novelty of this project is that we will avoid the usual backward-in-time adjoint solve. Instead, we will utilize a modern *flow-map* technique [4], which simplifies and enhances the efficiency of the adjoint procedure.

Who? This internship targets physicists or engineering science students in their master's with an understanding of numerical mathematics and who are comfortable with programming in Python/Matlab or C++.

Workplan:
The tasks for this project can be divided into the following points:

- Implementation of a classical adjoint framework as done in [3].
- Combination with existing framework [4].
- Fine-tuning the optimization for a predefined $f^{\exp}$

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References

[1] Fredi Tröltzsch. *Optimal control of partial differential equations: theory, methods and applications*, volume 112. American Mathematical Society, 2024.

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[3] Lukas Einkemmer, Qin Li, Li Wang, and Yang Yunan. Suppressing instability in a vlasov–poisson system by an external electric field through constrained optimization. *Journal of Computational Physics*, 498:112662, 2024.

[4] Philipp Krah, Xi-Yuan Yin, Julius Bergmann, Jean-Christophe Nave, and Kai Schneider. A characteristic mapping method for vlasov–poisson with extreme resolution properties. *Communications in Computational Physics*, 35(4):905–937, June 2024.